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THE TRIANGLE ANOMALY IN THE TRIPLE-REGGE LIMIT*

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Abstract

The U(1) triangle anomaly is present, as an infra-red divergence, in the six-reggeon triple-regge interaction vertex obtained from a maximally non-planar Feynman diagram in the full triple-regge limit of three-to-three quark scattering.

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1 Background Motivation

Our eventual goal [1] is to find a Reggeon Field Theory (RFT) description of the pomeron in QCD. For this the pomeron should be a Regge pole plus multi-pomeron exchanges and interactions - in agreement with experiment but not with perturbative QCD. A major motivation is that the RFT Critical pomeron [2] is the only known asymptotic solution of both s- and t- channel unitarity producing a rising total cross-section. In addition we also anticipate that the fundamental properties of confinement and chiral-symmetry breaking will be related to the regge behavior of the pomeron.

We have previously established [3] the existence of a "super-critical phase" of RFT which contains a regge pole pomeron but also contains both a reggeized massive vector (gluon?) partner for the pomeron and a "pomeron condensate". These are properties we might expect to find in a color superconducting phase of QCD in which SU(3) color is broken to SU(2). The symmetry-breaking vector mass would then be identified with the RFT order parameter and the critical pomeron would appear at the critical point where color superconductivity disappears and full SU(3) symmetry is restored. Also, in this identification, the condensate should carry the quantum numbers of the winding-number current suggesting, perhaps, that it is associated with spectral flow of the Dirac sea. This is why we look for a reggeon condensate arising from an "anomaly" infra-red divergence which we will eventually study in the dynamical context of the superconducting phase.

We must first demonstrate that it is possible for the anomaly to actually appear in a regge-limit effective theory. It is, of course, absent in the usual perturbation expansion of a vector theory. This talk will concentrate on this issue. The consequences (RFT for the pomeron, confinement, etc.) will be discussed only very briefly. An extended version of the analysis we outline, together with other closely related analyses, can be found elsewhere [4].

2 A New Manifestation of the U(1) Anomaly.

We study 3-3 quark scattering in a "triple-regge" limit [5] involving three large light-cone momenta and consider diagrams containing a single quark loop.

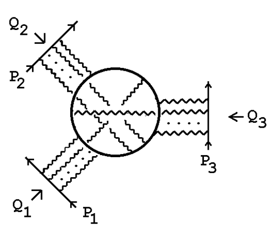

Fig. 1 3-3 Quark Scattering

$$\begin{aligned} P_1 &\rightarrow P_1^+ = (p_1, p_1, 0, 0)\ , \quad p_1 \rightarrow \infty \\ P_2 &\rightarrow P_2^+ = (p_2, 0, p_2, 0)\ , \quad p_2 \rightarrow \infty \\ P_3 &\rightarrow P_3^+ = (p_3, 0, 0, p_3)\ , \quad p_3 \rightarrow \infty \\ Q_1 &\rightarrow (\hat{q}_1, \hat{q}_1, q_{12}, q_{13}) \\ Q_2 &\rightarrow (\hat{q}_2, q_{21}, \hat{q}_2, , q_{23}) \\ Q_3 &\rightarrow (\hat{q}_3, q_{31}, q_{32}, \hat{q}_3) \end{aligned}$$

Using light-cone co-ordinates the leading behaviour (up to logarithms) is obtained by putting quark lines on-shell via $k_{i\pm}$ integrations. As illustrated in Fig. 2 the resulting

"reggeon diagrams" [1] are $k_\perp$-integrals in which the "reggeon interactions" are quark triangle diagrams with local (and non-local) "effective vertices" containing γ-matrix products which (in some cases) give the $\gamma_5\gamma_\mu$ couplings that could produce the triangle anomaly. In particular the "maximally non-planar" diagram of Fig. 3 gives a six-reggeon interaction as shown.

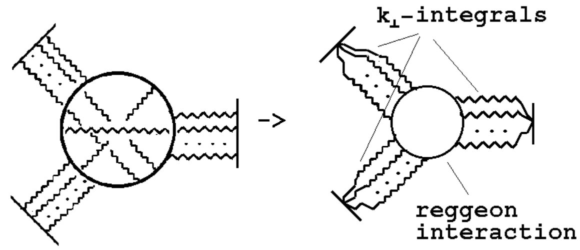

Fig. 2 Reggeon Diagram Generation.

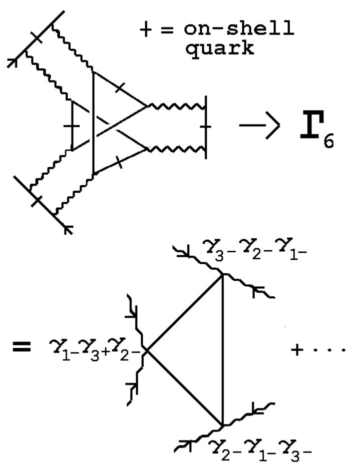

Fig. 3 Six-Reggeon Interaction

At lowest-order (two gluons in each Q_i channel) there are ~ 100 diagrams that potentially give contributions. To systematically evaluate all contributions and to discuss cancelations, it is necessary to develop a multi-regge asymptotic dispersion relation formalism [6] in which multiple discontinuities are initially calculated rather than amplitudes. Using this formalism it can be shown [4] that the anomaly occurs only in the maximally non-planar diagrams. In this talk we will not discuss the dispersion relation formalism at all but rather concentrate on showing how the anomaly occurs in such diagrams.
To avoid ultra-violet subtleties we look for the anomaly in the infra-red region.

3 The Anomaly as an Infra-red Divergence.

For a massless quark axial-vector current the anomaly divergence equation for the three current vertex $\Gamma_{\mu\nu\lambda}(\mathbf{q}_1, \mathbf{q}_2, \mathbf{q}_1 + \mathbf{q}_2)$ implies [7]

$$\Gamma_{\mu\nu\lambda} \sim \epsilon_{\mu\nu\alpha\beta} \frac{\mathbf{q}_1^\lambda \mathbf{q}_1^\alpha \mathbf{q}_2^\beta}{\mathbf{q}_1^2} + \cdots , \qquad \mathbf{q}_1^2 \sim \mathbf{q}_2^2 \sim (\mathbf{q}_1 + \mathbf{q}_2)^2 \sim \mathbf{q}^2 \quad \to 0 \tag{1}$$

For the U(1) current we are, of course, ignoring non-perturbative contributions. We will be looking for a "perturbative" effect! If $\mathbf{q}_{1+} \not\to 0$, and $\mathbf{q}_2$ is spacelike with $\mathbf{q}_2 \perp \mathbf{q}_{1+}$ then

$$\epsilon_{\mu\nu\alpha\beta}\ \mathbf{q}_1^\lambda \mathbf{q}_1^\alpha \mathbf{q}_2^\beta / \mathbf{q}^2 \ \sim\ {\mathbf{q}_{1+}}^2/\mathbf{q} \ \sim\ 1/\mathbf{q} \tag{2}$$

For our purposes, this linear divergence characterises the anomaly since we will not have a Lorentz-covariant amplitude that separates into kinematic and invariant factors. The divergence is produced by the triangle diagram Landau singularity and the light-like momentum $\mathbf{q}_{1+}$ is essential. It can only be canceled by other quark triangle contributions.

4 A Contribution from a Maximally Non-planar Diagram.

We label loop momenta as in Fig. 4. For the k_1 and k_2 integrations we use (new) light-cone co-ordinates [4]

$$k_i = k_{i2-}\underline{n}_{1+} + k_{i1-}\underline{n}_{2+} + \underline{k}_{i12+}$$

$i = 1, 2$, together with conventional light-cone co-ordinates

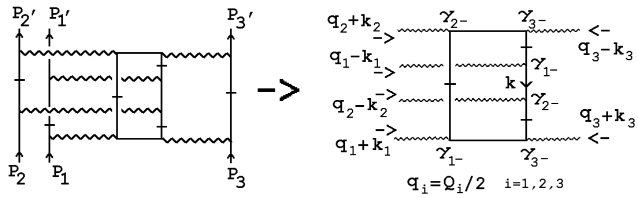

Fig. 4 Loop Momenta

$(k_{3-}, k_{3-}, \tilde{\underline{k}}_{3\perp})$ for the k_3 integration. The k_{11-}, k_{22-}, k_{3-} integrations are straightforward. There are six options for using the remaining longitudinal k_i integrations to put hatched lines on-shell. We look for the anomaly to be generated by a combination of local γ-matrix couplings, with an odd number of γ_5's. Only one option gives couplings, all three of which are local. A local coupling is obtained from that part of a quark numerator with the same momentum factor that scales the integrated longitudinal momentum, e.g.

$$\begin{aligned} &\int dk_{12-}\,\delta\Big((k_1 + k - q_1)^2 - m^2\Big)\gamma_{3-}\Big((k_1 + k - q_1)\cdot\gamma + m\Big)\gamma_{1-} \\ &= \int dk_{12-}\,\delta\Big(k_{1-}\ k_{12-} + \cdots\Big)\gamma_{3-}\Big(k_{1-}\cdot\gamma_{2-} + \cdots\Big)\gamma_{1-} \ = \ \gamma_{3-}\gamma_{2-}\gamma_{1-} \ + \ \cdots \end{aligned} \tag{3}$$

That part of the asymptotic amplitude with local couplings is

$$\begin{aligned} &g^{12}\ \frac{p_1\ p_2\ p_3}{m^3} \qquad \times \\ &\int \frac{d^2\underline{k}_{112+}}{(q_1 + \underline{k}_{112+})^2(q_1 - \underline{k}_{112+})^2} \int \frac{d^2\underline{k}_{212}}{(q_2 + \underline{k}_{212+})^2(q_2 - \underline{k}_{212+})^2} \int \frac{d^2\underline{k}_{33\perp}}{(q_3 + \underline{k}_{33\perp})^2(q_3 - \underline{k}_{33\perp})^2} \\ &\int d^4k\ \frac{Tr\{\hat{\gamma}_{12}(\not{k} + \not{k}_1 + \not{q}_2 + \not{k}_3 + m)\hat{\gamma}_{31}(\not{k} + m)\hat{\gamma}_{23}(\not{k} - \not{k}_2 + \not{q}_1 + \not{k}_3 + m)\}}{([k + k_1 + q_2 + k_3]^2 - m^2)(k^2 - m^2)([k - k_2 + q_1 + k_3]^2 - m^2)} \end{aligned} \tag{4}$$

where $k_{11-} = k_{22-} = k_{33-} = 0$, k_{12-}, k_{21-} and k_{33+} are determined by mass-shell δ-functions, and

$$\begin{aligned} \hat{\gamma}_{31} &= \gamma_{3-}\gamma_{2-}\gamma_{1-} = \gamma_{-,+,-} - i\,\gamma_5\,\gamma_{-,-,-} \\ \hat{\gamma}_{23} &= \gamma_{2-}\gamma_{1-}\gamma_{3-} = \gamma_{+,-,-} - i\,\gamma_5\,\gamma_{-,-,-} \\ \hat{\gamma}_{12} &= \gamma_{1-}\gamma_{3+}\gamma_{2-} = \gamma_{-,-,-} + i\,\gamma_5\,\gamma_{-,-,+} \end{aligned} \tag{5}$$

with $\gamma_{\pm,\pm,\pm} = \gamma\cdot n_{\pm,\pm,\pm}$, $n_{\pm,\pm,\pm} = (1, \pm1, \pm1, \pm1)$.

Removing the transverse momentum integrals and gluon propagators as the lowest-order contributions of two-reggeon states in each t_i-channel, the three γ_5 couplings give the $m = 0$

reggeon interaction

$$\Gamma_6(q_1,q_2,q_3,\tilde{\underline{k}}_1,\tilde{\underline{k}}_2,\underline{k}_{3\perp},0) \;=\; \int d^4k \frac{Tr\{\gamma_5\gamma_{-,-,+}(\not{k}+\not{k}_1+\not{q}_2+\not{k}_3)\gamma_5\gamma_{-,-,-}\;\not{k}\;\gamma_5\gamma_{-,-,-}(\not{k}-\not{k}_2+\not{q}_1+\not{k}_3)}{(k+k_1+q_2+k_3)^2\;k^2\;(k-k_2+q_1+k_3)^2} \tag{6}$$

This interaction corresponds to the triangle diagram of Fig. 5. As discussed above, to obtain the maximal anomaly infra-red divergence we must have a component of the axial-vector triangle diagram tensor $\Gamma^{\mu\nu\lambda}$ with $\mu=\nu$ having a lightlike projection and λ an orthogonal index having a spacelike projection. This requirement is met if the light-like projection is made on either $\underline{n}_{1+}$ or $\underline{n}_{2+}$. In each case, $\gamma^{-,-,+}$ has the necessary orthogonal spacelike component. The anomaly divergence appears if we can take the limit

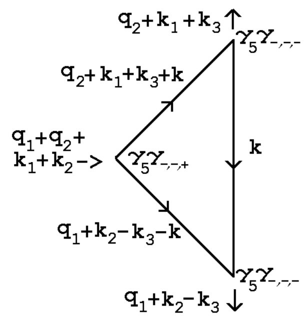

Fig. 5 A Triangle Diagram.

$$(k_1+q_2+k_3)^2 \sim (q_1+q_2+k_1+k_2)^2 \sim (k_2+q_1-k_3)^2 \sim \mathbf{q}^2 \to 0 \tag{7}$$

while keeping a finite light-like momentum, parallel to either $\underline{n}_{1-}$ or $\underline{n}_{2+}$, flowing through the diagram. The mass-shell constraints must also be satisfied.

We first consider $\mathbf{q}=0$ with the loop momentum $k\sim\mathbf{q}=0$. It is then straightforward to add momenta that are $O(\mathbf{q})$. If we impose

$$q_1+q_2+k_1+k_2 \;=\; 0\;, \qquad q_1+k_2-k_3 \;=\; (2l,-2l,0,0) \;\;\sim\;\; \underline{n}_{1-} \tag{8}$$

and take q_1-k_1 and q_2-k_2 lightlike with

$$q_{12-} = -k_{12-} = l\;,\quad q_{21-} = -k_{21-} = -l\;,\qquad \underline{q}_{112+} = \underline{k}_{112+} = -\underline{q}_{212+} = -\underline{k}_{212+} \tag{9}$$

then $q_3=-q_1-q_2=(0,l,-l,0)$ has the necessary spacelike form and if $k_3=(0,3l,l,0)$ with

$$8lq_{112-} \;=\; q_1^2 \;=\; \underline{q}_{112+}^2 \;=\; q_{112-}^2+q_{13}^2 \tag{10}$$

then all requirements are satisfied and

$$(q_1+k_1)^2 \;=\; 4q_1^2 \;=\; (q_2+k_2)^2 \;=\; 4q_2^2 \;=\; 32lq_{112-} \;=\; \frac{32}{\sqrt{2}}(q_3^2)^{1/2}(q_1^2-q_{13}^2)^{1/2} \tag{11}$$

Adding momenta $O(\mathbf{q})$ parallel to $\underline{n}_{12-}$, the limit $\mathbf{q}\to 0$, with l fixed, gives

$$(q_1-k_1)^2 \;\sim\; (q_2-k_2)^2 \;\sim\; \mathbf{q}^2 \;\to 0 \tag{12}$$

while

$$(q_1+k_1)^2 \;\to\; (q_2+k_2)^2 \not\to 0\;,\quad (q_3-k_3)^2 \;\to\; 2(q_3+k_3)^2 \not\to 0 \tag{13}$$

and the six-reggeon vertex Γ_6 has the divergence

$$\Gamma_6 \;\sim\; \epsilon_{\underline{n}_{1+},\underline{n}_3,\underline{n}_{2+},\underline{n}_{12+}} \;\frac{l^2}{\mathbf{q}} \;\sim\; \frac{Q_3^2}{\mathbf{q}} \tag{14}$$

If we instead impose

$$q_2 + k_1 + k_3 \;=\; (2l, 0, -2l, 0) \;\;\sim\; \underline{n}_{2-} \tag{15}$$

the role of 1 and 2 is interchanged together with $k_3 \leftrightarrow -k_3$. Although $Q_1^2 = Q_2^2$ in both cases, the antisymmetry of the ϵ-tensor does not produce a cancelation of the two divergences within the reggeon vertex Γ_6. However, in the full hatched diagram of Fig. 4 the k_3 integration is symmetric and there will be a cancelation after integration.

In higher-orders the two gluons in each t_i channel are replaced by even signature two-reggeon states with couplings G_1, G_2 and G_3 to the external scattering states and the integration in individual diagrams is not necessarily symmetric. However, if we keep the same quark loop interaction, the amplitude with even signature in each channel is obtained by summing over all diagrams related by a twist ($P_i \leftrightarrow P_{i'},\ i = 1, 2, 3$). Both the color factors and all three of the k_i-integrations are then symmetric and the anomaly cancels [4]. Sufficient antisymmetry to avoid cancelation would appear only if all three reggeon states had anomalous color parity ($\neq$ signature). However, such reggeon states do not couple to elementary scattering quarks (or gluons),

In more complicated scattering processes anomaly interactions can appear without cancelation. However, all the reggeon states involved carry anomalous color parity [4]. Such states appear when additional particles are produced (or absorbed) at the external vertices. Since anomaly vertices do not conserve color parity they have to appear pairwise. Also, while there is an infra divergence in the reggeon interaction in which it appears, the anomaly not produce divergences in full amplitudes because of compensating reggeon Ward identity zeroes [1].

If some external couplings are chosen so that particular reggeon Ward identity zeroes are absent, the anomaly does produce divergent amplitudes. A reggeon condensate with the quantum numbers of the winding-number current can be introduced this way. In [1] it was argued that in the color superconducting supercritical phase this condensate is consistently reproduced in all reggeon states by anomaly infra-red divergences, while also producing confinement, chiral symmetry breaking and a regge pole pomeron. Having the full structure of the anomaly under control, we hope to implement the program of [1] in detail in future papers. In effect we aim to to show that in massless QCD infra-red divergences produce a transition from perturbative reggeon diagrams to diagrams containing hadrons and the pomeron. If we obtain a unitary (reggeon) S-Matrix, as we anticipate, it will be very close to perturbation theory, with the non-perturbative properties of confinement and chiral symmetry breaking a consequence of the anomaly only.

References

1. A. R. White, *Phys. Rev.* **D58**, 074008 (1998), see also Lectures in the Proceedings of the Theory Institute on Deep-Inelastic Diffraction, Argonne National Laboratory (1998).
2. A. A. Migdal, A. M. Polyakov and K. A. Ter-Martirosyan, *Zh. Eksp. Teor. Fiz.* **67**, 84 (1974); H. D. I. Abarbanel and J. B. Bronzan, *Phys. Rev.* **D9**, 2397 (1974).
3. A. R. White, Int. J. Mod. Phys. **A11**, 1859 (1991).
4. A. R. White, hep-ph/9910458, an abbreviated version of the main calculation is given in hep-ph/9910459.
5. P. Goddard and A. R. White, *Nucl. Phys.* **B17**, 1, 45 (1970).
6. H. P. Stapp and A. R. White, *Phys. Rev.* **D26**, 2145 (1982); H. P. Stapp in *Structural Analysis of Collision Amplitudes*, proceedings of the Les Houches Institute, eds. R. Balian and D. Iagolnitzer (North Holland, 1976).
7. S. Coleman and B. Grossman, *Nucl. Phys.* **B203**, 205 (1982).